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# Born-Infeld Actions from Matrix Theory

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## Abstract:

We propose a formula for the effective action of Matrix Theory which successfully reproduces a large class of Born-Infeld type D-brane probe actions. The formula is motivated by demanding consistency with known results, and is tested by comparing with a wide range of source-probe calculations in supergravity. In the case of D0-brane sources and Dp-brane probes, we study the effect of boosts, rotations, and worldvolume electric fields on the probe, and find agreement with supergravity to all orders in the gravitational coupling. We also consider D4-brane sources at the one loop level and recover the correct probe actions for a D0-brane, and for a D4-brane rotated at an angle with respect to the source.

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# 1 Introduction

A remarkable feature of Matrix Theory (MT) [1] is its ability to describe a wide variety of objects within a single configuration space; the various known branes of string theory can be realized by choosing specific forms for the MT variables<sup>1</sup>. One can also realize two widely separated branes, and study the resulting interactions by integrating out the massive degrees of freedom which couple the objects [3, 4, 5, 6, 7]. The results so obtained can be compared with the interactions predicted by supergravity, and can thus be used as checks of the MT proposal. During the past year, numerous checks of this sort have been performed, and have lent strong support to the accuracy of MT [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. Typically, one computes one loop diagrams in Matrix quantum mechanics and compares the result against the leading long-distance interaction from supergravity. In the case of D0-brane scattering, one thereby verifies the coefficient of the well known  $v^4/r^7$  term. In addition – and even more impressively – there have been calculations at the two loop level [23, 24] (see also [25]) which have reproduced the subleading  $v^6/r^{14}$  interaction .

On the supergravity side, the D0-D0 scattering amounts to studying the D0-brane probe action in the presence of background fields corresponding to a D0-brane source and obtained by null reduction from eleven dimensions [24, 26]. This action gives predictions for terms of the form  $v^{2l+1}/r^{7l}$ ;  $l = 0, 1, 2, \dots$  and reads

$$S_0 = -T_0 \int dt \, h^{-1} [\sqrt{1 - hv^2} - 1] ,$$

where  $h = Q_0/r^7$ , and  $T_0, Q_0$  are the D0-brane's mass and charge, respectively. To explicitly verify that  $S$  is reproduced in MT, one would have to carry out the highly demanding computation of three loop and higher diagrams. In this work we adopt a different approach: we will assume that  $S$  is in fact properly reproduced, use this assumption to propose a general form for the MT effective action, and then test our proposal by using the action to compute the interactions of other types of branes. The point is that the same action which leads to  $S_0$  when the background describes D0-branes must, by consistency, also yield the correct action for, say, a D2-brane and a D0-brane, when the background is changed accordingly. These consistency restrictions are so demanding that they plausibly prescribe a unique form for the effective action governing a wide class of backgrounds.

The sorts of backgrounds we are interested in, those describing two separated objects, are given by the matrices

$$X_i = \begin{pmatrix} (\mathcal{U}_i)_{N_1 \times N_1} & \\ & (\mathcal{V}_i)_{N_2 \times N_2} \end{pmatrix} ; \quad i = 1 \dots 9$$

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<sup>1</sup>There is, however, difficulty in realizing the transverse five brane, as Matrix theory appears to lack the needed central charge [2].

where  $\mathcal{U}_i, \mathcal{V}_i$  are themselves matrices of variable size. To write our ansatz for the effective action we need a few definitions. We define a field strength tensor  $\mathcal{F}_{MN}$  by:

$$\mathcal{F}_{0i} = \dot{\mathcal{U}}_i \otimes \mathbf{1} - \mathbf{1} \otimes \dot{\mathcal{V}}_i^* \quad (1)$$

$$\mathcal{F}_{ij} = -i[\mathcal{U}_i, \mathcal{U}_j] \otimes \mathbf{1} + \mathbf{1} \otimes i[\mathcal{V}_i^*, \mathcal{V}_j^*] \quad (2)$$

Next, we define a “harmonic function”  $h$  by

$$h = Q_0 ([\mathcal{U}_i \otimes \mathbf{1} - \mathbf{1} \otimes \mathcal{V}_i^*][\mathcal{U}_i \otimes \mathbf{1} - \mathbf{1} \otimes \mathcal{V}_i^*])^{-7/2} . \quad (3)$$

With these definitions in hand, our proposal for the effective action reads

$$S_{MT} = -T_0 \text{Tr} \int dt h^{-1} \{ \sqrt{-\det[\eta_{MN} - h^{1/2} \mathcal{F}_{MN}]} - 1 \} , \quad (4)$$

where  $\eta_{MN} = \text{diag}(1, -1, -1, \dots, -1)$ , and  $\text{Tr}$  means a particular trace operation over  $\mathcal{U}, \mathcal{V}$  space to be discussed in the next section. It is simple to check (see section 3) that  $S_{MT}$  reduces to  $S_0$  when  $\mathcal{U}_i, \mathcal{V}_i$  are chosen to be D0-brane backgrounds. We should emphasize that  $S_{MT}$  is not meant to be the full effective action of Matrix Theory, but rather a portion of it which is valid under restricted conditions. For instance, important simplifications occur when the field strength commutes:

$$[\mathcal{F}_{\mu\nu}, \mathcal{F}_{\alpha\beta}] = [\mathcal{F}_{\mu\nu}, \mathcal{U}_i \otimes \mathbf{1}] = [\mathcal{F}_{\mu\nu}, \mathbf{1} \otimes \mathcal{V}_i] = 0 ;$$

we will refer to this case as “abelian”. If the background is not abelian in this sense, there will be corrections to  $S_{MT}$  depending on commutators. (See [27] for a related discussion). Nevertheless,  $S_{MT}$  is sufficiently general that it encompasses a wide range of source-probe type calculations.

The remainder of this paper is devoted to motivating and testing  $S_{MT}$  for a variety of backgrounds. In section 2 we review some basic principles and establish conventions. Our ansatz for  $S_{MT}$  is motivated in section 3 by recalling the cases where the loop diagrams have been computed explicitly, and demanding consistency with those results. We perform a number of detailed checks of  $S_{MT}$  in section 4 by studying the action of a D2-brane probe in the presence of a D0-brane source. We consider rotating the D2-brane and show that the action changes in the correct way to agree with supergravity. In section 5 we consider replacing the D2-brane probe by a general D2p-brane probe, and show that the agreement with supergravity persists. We also include transverse velocities and worldvolume electric fields on the probe. In section 6 we replace the D0-brane source by a D4-brane source. For reasons to be discussed, our checks in this case are restricted to the one loop level. We study the case of a D0-brane probe as well as a D4-brane probe oriented at an angle relative to the source. Again, we find agreement. In section 7 we review our results and discuss the prospects for extending our methods to more general backgrounds. Finally, an Appendix contains the derivations of D0-brane and D4-brane supergravity backgrounds via null reduction from eleven dimensions.

## 2 Conventions

In string units (we set  $2\pi\alpha' = 1$ ) the Matrix Theory action is [1]

$$S = \frac{T_0}{2} \int dt \operatorname{Tr} \{ (D_t X_i)^2 + \frac{1}{2} [X_i, X_j]^2 + 2\Theta^T \dot{\Theta} - 2\Theta^T \gamma_i [\Theta, X^i] \} \quad (5)$$

where  $i, j = 1, \dots, 9$ ,  $D_t = \partial_t - i[A_t, \cdot]$  and  $T_0$  is the mass of a D0-brane. We will be considering various backgrounds for the bosonic fields  $X_i$ . According to Susskind's DLCQ proposal [26], when the  $X_i$  are  $N \times N$  matrices, one is studying M theory with a null direction compactified and with  $N$  units of longitudinal momentum.

The background corresponding to  $N$  D0-branes is

$$X_i = \begin{pmatrix} x_i^{(1)} & & & \\ & x_i^{(2)} & & \\ & & \ddots & \\ & & & x_i^{(N)} \end{pmatrix}$$

where  $x_i^{(a)}$  represents the position of the  $a$ th D0-brane.

To construct the background corresponding to a D2-brane [28, 1], one introduces the canonical variables  $Q, P$  satisfying  $[Q, P] = i$ , which are thought of as matrices of infinite size. The trace operation over  $Q, P$  space is given by

$$\operatorname{Tr} \rightarrow \frac{1}{2\pi} \int dP dQ. \quad (6)$$

In Matrix Theory, one considers not a pure D2-brane, but rather a bound state of D0-branes and a D2-brane – the so called (2+0) configuration. The density  $\sigma_0$  of D0-branes on the D2-brane is described in terms of a magnetic field [29, 30]:  $\sigma_0 = F_{12}/2\pi$ , where we take the brane to lie in the (12)-plane. The MT background for such a configuration is

$$X_1 = \frac{Q}{\sqrt{F_{12}}} ; X_2 = \frac{P}{\sqrt{F_{12}}} ; X_{I>2} = 0 . \quad (7)$$

The division by  $\sqrt{F_{12}}$  gives the D2-brane the correct central charge in the D=11 SUSY algebra [2]. The above background represents the “minimal” (2+0) configuration, to which one can add electric fields, boosts, rotations, and non-extremal excitations. In subsequent sections we'll describe how these variables are represented in Matrix Theory.

Other Dp-branes are represented by essentially repeating the above procedure. For instance, a four brane in the (1234)-hyperplane is given by

$$X_1 = \frac{Q_1}{\sqrt{F_{12}}} ; X_2 = \frac{P_1}{\sqrt{F_{12}}} ; X_3 = \frac{Q_2}{\sqrt{F_{34}}} ; X_4 = \frac{P_2}{\sqrt{F_{34}}} . \quad (8)$$

This object is a (4+2+2+0) configuration consisting of a D4-brane, D2-branes in the (12)-plane with density  $F_{12}/2\pi$ , D2-branes in the (34)-plane with density  $F_{34}/2\pi$ , and D0-branes with density  $F_{12}F_{34}/(2\pi)^2$ . One can also construct D4-brane configurations with vanishing D2-brane density [31], as we discuss in section 6.

Now let us turn to the supergravity description of D-brane interactions. A source D0-brane is described by the field configuration

$$\begin{aligned} ds^2 &= h^{1/2} dt^2 - h^{-1/2} (dx_1^2 + \dots + dx_9^2) \\ e^{-\phi} &= h^{-3/4} \quad ; \quad C_t^{(1)} = h^{-1} \\ h &= \frac{Q_0}{r^7} . \end{aligned} \tag{9}$$

The solution above is not the standard D0 solution of IIA supergravity, but instead comes from the null reduction of a graviton solution in D=11, as reviewed in the Appendix. The primary difference with the standard solution is that normally  $h = 1 + (Q_0/r^7)$  but here the 1 is absent. As discussed in [24], this form for  $h$  is crucial to obtain agreement with Matrix Theory.

The interaction of the D0-brane with a Dp-brane will be described by treating the Dp-brane as a probe. The action for the probe is the Born-Infeld action with a Chern-Simons term [32, 30]:

$$S_p = -T_p \int d^{p+1} \xi \left\{ e^{-\phi} \sqrt{\det[g_{MN} \partial_\mu X^M \partial_\nu X^N - F_{\mu\nu}]} - \frac{1}{2^{p/2} (p/2)!} \epsilon_{i_1 \dots i_p} F_{i_1 i_2} \dots F_{i_{p-1} i_p} C_t^{(1)} \right\} , \tag{10}$$

where  $T_p = T_0/(2\pi)^{(p/2)}$  is the tension of the  $p$ -brane. The indices  $M, N$  are spacetime indices in 10 dimensions, the indices  $\mu, \nu$  are indices in the  $p+1$  dimensional worldvolume, and the indices  $i_1, i_2, \dots$  are spacelike indices in the worldvolume. The above probe action is valid for fields which are slowly varying (on a length scale set by the string scale) on the worldvolume; for rapidly varying fields one expects derivative corrections. By plugging in the background fields of the D0-brane one obtains an effective action for the Dp-brane which governs its interaction with the D0-brane to all orders in the string coupling. One of our goals is to show how such an action can arise from Matrix Theory.

### 3 The general action for two-body interactions

Let us consider the Matrix Theory (MT) background for two arbitrary separated objects

$$(X_i)_{(N_1+N_2) \times (N_1+N_2)} = \begin{pmatrix} (\mathcal{U}_i)_{N_1 \times N_1} & \\ & (\mathcal{V}_i)_{N_2 \times N_2} \end{pmatrix} . \tag{11}$$

Interactions between objects  $\mathcal{U}$  and  $\mathcal{V}$  arise by expanding the Matrix Theory action around this background and integrating out the massive degrees of freedom to a given number of loops. In general, one expects to obtain a horribly complicated non-linear function of  $\mathcal{U}_i$  and  $\mathcal{V}_i$ . However, if attention is restricted to a subset of the diagrams, we'll argue that the terms sum up to give a simple form for the action. To motivate this formula let us first review the cases in which the loop calculations have been performed explicitly.

First consider the interaction of two D0-branes [3, 4, 5]. We take  $\mathcal{U}_i = (b_i + v_i t) \cdot \mathbf{1}_{N_1 \times N_1}$ ;  $\mathcal{V}_i = \mathbf{0}_{N_2 \times N_2}$ . In [24] it was shown that the two loop effective action coincides with the result of expanding

$$S_0 = N_1 T_0 \int dt h^{-1} [\sqrt{1 - h v^2} - 1] \quad ; \quad h = \frac{N_2 Q_0}{b^7} \quad (12)$$

to order  $h^2$ .  $S_0$  is the action of a probe D0-brane moving in the background of a D0-brane source. The loop expansion of Matrix Theory corresponds to the  $h$  expansion of supergravity, so if MT is correct, the infinite series of higher loop diagrams should reproduce the full form for  $S_0$ . It is of course important to check explicitly that this is indeed the case, but here we will simply assume that it is true.

In fact the MT action contains many more terms than those displayed in (12) [24, 33]. An important point concerns the  $N_1$  and  $N_2$  dependence of  $S_0$ ; specifically,  $S_0$  is linear in  $N_1$ . In general, an  $l$  loop diagram yields terms of the form  $N_1^\alpha N_2^\beta$  with  $\alpha + \beta = l + 1$ . However, the source-probe calculations in supergravity that we are comparing with only correspond to terms linear in the number of D0-branes of the probe, thus we keep only those terms in MT as well. In a similar vein, the  $v$  and  $b$  dependences of the full MT action will differ from what appears in  $S_0$ . In particular, the  $l$  loop diagrams can yield  $b^{4-3l} f(v^2/b^4)$  whereas  $S_0$  yields only terms like  $v^{2(l+1)}/b^{7l}$ . The extra terms in MT correspond to effects not included in our supergravity calculations – such as the presence of  $\mathcal{R}^4$  terms [33, 34, 35] in the gravitational action – so we drop them. To summarize: by keeping terms in the MT action which are linear in  $N_1$  and of the form  $v^{2(l+1)}/b^{7l}$ , we expect to find complete agreement with the probe action from supergravity.

To obtain more information we turn to backgrounds which are more general, but for which only one loop results are known. Ref. [17] considered the background

$$X_i = \begin{pmatrix} (\mathcal{U}_i)_{N_1 \times N_1} & \\ & (\mathcal{V}_i)_{N_2 \times N_2} \end{pmatrix} \quad i = 1 \dots 7 \quad (13)$$

$$X_8 = \begin{pmatrix} b \cdot \mathbf{1}_{N_1 \times N_1} & \\ & \mathbf{0}_{N_2 \times N_2} \end{pmatrix} \quad (14)$$

$$X_9 = \begin{pmatrix} vt \cdot \mathbf{1}_{N_1 \times N_1} & \\ & \mathbf{0}_{N_2 \times N_2} \end{pmatrix} \quad (15)$$

and evaluated the one-loop determinants for arbitrary static and “abelian”  $\mathcal{U}_i, \mathcal{V}_i$ . The determinants can be converted into an effective action, which reads:

$$S_{1\text{-loop}} = -\frac{1}{8}T_0 \text{Tr}^{(N_1)} \text{Tr}^{(N_2)} \int dt \, \mathbf{h} \left[ \text{tr}(\eta \mathcal{F})^4 - \frac{1}{4}(\text{tr}(\eta \mathcal{F})^2)^2 \right] \quad (16)$$

Here,  $\text{tr}$  refers to a trace over Lorentz indices.

We wish to write down an ansatz for the MT effective action which reproduces the special cases just described. Given this constraint, there is a very natural guess for the action:

$$S_{MT} = -T_0 \text{Tr}^{(N_1)} \hat{\text{Tr}}^{(N_2)} \int dt \, \mathbf{h}^{-1} \{ \sqrt{-\det[\eta_{MN} - \mathbf{h}^{1/2} \mathcal{F}_{MN}]} - 1 \} \quad (17)$$

The  $\hat{\text{Tr}}^{(N_2)}$  operator requires some explanation, which we provide with reference to double line notation. In this notation, an  $l$  loop planar diagram contributing to the action consists of an outer loop associated with an index in  $\mathcal{U}_i$  space, and  $l$  closed inner loops associated with an index in  $\mathcal{V}_i$  space. Each loop gives rise to a trace in the corresponding space, so an  $l$  loop diagram gives, schematically,  $\text{Tr}^{(N_1)}(\text{Tr}^{(N_2)})^l$ . Each loop contains a number of insertions of the background field and these appear inside the traces. However, there are various ways of partitioning the background field operators among the various loops and this leads to a number of distinct diagrams at a given loop order. Consider, for example, the two loop (order  $\mathbf{h}^2$ ) contribution:

$$S_{2\text{-loop}} \sim \text{Tr}^{(N_1)} \text{Tr}^{(N_2)} \text{Tr}^{(N_2)} \mathbf{h}^2 \mathcal{F}^6 \quad (18)$$

This expression could stand for (schematically)

$$\text{Tr}^{(N_1)} \{ \text{Tr}^{(N_2)} (\mathbf{h}^2 \mathcal{F}^6) \text{Tr}^{(N_2)} (\mathbf{1}_{N_2 \times N_2}) + \text{Tr}^{(N_2)} (\mathbf{h} \mathcal{F}^4) \text{Tr}^{(N_2)} (\mathbf{h} \mathcal{F}^2) + \dots \} \quad (19)$$

Without explicitly evaluating the diagrams, there is an ambiguity as to the relative weighting of the different terms, and the ambiguity only gets worse for higher orders in  $\mathbf{h}$ . Fortunately, there are two important cases for which the ambiguity is absent. The first case arises when we consider D0-brane sources:  $\mathcal{V}_i = \mathbf{0}_{N_2 \times N_2}$ . In such cases  $\mathbf{h}$  and  $\mathcal{F}$  are proportional to  $\mathbf{1}_{N_2 \times N_2}$  and so it does not matter how the traces are distributed: every way gives  $N_2^l$ . Thus for D0-brane sources the  $\hat{\text{Tr}}^{(N_2)}$  operator is equal to  $N_2^l$  when acting on a term of order  $\mathbf{h}^l$ . Therefore, for D0-brane sources, we should be able to compare to all orders our ansatz for the action against the supergravity prediction for an arbitrary brane – extremal or non-extremal – moving in the background of a D0-brane. We will perform these checks in the next sections.

The other unambiguous case occurs when we restrict attention to 1-loop results. In this case  $\hat{\text{Tr}}^{(N_2)} \rightarrow \text{Tr}^{(N_2)}$ , *i.e.* just a single standard trace. This time we can consider

other sources besides D0-branes, and the results should still match with supergravity. We check this in section 6 by considering D4-brane sources. At the one loop level our proposed action can be proven directly by evaluating the relevant Feynman diagrams, but at this time this has only been done for various special cases. Note that aspects of the one loop correspondence between supergravity and MT have previously been discussed in [18].

As a first test of our ansatz, we will verify that it indeed reproduces (12) and (16). First consider the D0-D0 case. In this example

$$\mathcal{F}_{0i} = v_i ; \mathcal{F}_{ij} = 0 ; \mathbf{h} = \frac{Q_0}{|\vec{b} + \vec{v}t|^7} \quad (20)$$

and

$$\det[\eta_{MN} - \mathbf{h}^{1/2}\mathcal{F}_{MN}] = -(1 - \mathbf{h}v^2) . \quad (21)$$

So,

$$\begin{aligned} S_{MT} &= -T_0 \text{Tr}^{(N_1)} \hat{\text{Tr}}^{(N_2)} \int dt \mathbf{h}^{-1} \{ \sqrt{1 - \mathbf{h}v^2} - 1 \} \\ &= -N_1 T_0 \int dt (N_2 \mathbf{h})^{-1} \{ \sqrt{1 - N_2 \mathbf{h}v^2} - 1 \} \\ &= -N_1 T_0 \int dt h^{-1} \{ \sqrt{1 - hv^2} - 1 \} , \end{aligned}$$

as desired. The other special case (16) follows immediately upon expanding to order  $\mathbf{h}$ :

$$\mathbf{h}^{-1} \sqrt{-\det[\eta_{MN} - \mathbf{h}^{1/2}\mathcal{F}_{MN}]} = \mathcal{O}(\mathbf{h}^0) + \frac{\mathbf{h}}{8} \left[ \text{tr}(\eta\mathcal{F})^4 - \frac{1}{4}(\text{tr}(\eta\mathcal{F})^2)^2 \right] + \mathcal{O}(\mathbf{h}^2) .$$

Now we turn to more demanding checks of the ansatz.

## 4 $0_s - (2+0)_p$ brane interactions

In this section we will check our ansatz for the Matrix Theory action within the context of the D0-brane, D(2+0)-brane system. After treating the simplest case, we will study the effect of rotating the (2+0) brane and recover actions which agree with supergravity to all orders.

The supergravity result which we will try to reproduce from Matrix Theory comes from the Born-Infeld action for the (2+0) probe. We use the action (10) with  $p = 2$ :

$$S_2 = -\frac{T_0}{2\pi} \int d^3\xi \left\{ e^{-\phi} \sqrt{\det[g_{MN} \partial_\mu X^M \partial_\nu X^N - F_{\mu\nu}]} - F_{12} C_t^{(1)} \right\} , \quad (22)$$

where  $g_{MN}, e^{-\phi}, C_t^{(1)}$  are the fields of a D0-brane given in (9).



## 4.1 Simplest case

We first consider the simplest case of a stationary brane with vanishing electric fields  $F_{0i}$ . We will choose the static gauge for the probe, which consists of setting the worldvolume coordinates  $\xi^{0,1,2}$  to be equal to the spacetime coordinates  $X^{0,1,2}$ ,

$$X^{0,1,2} = \xi^{0,1,2} \quad ; \quad X^3 = b_3 \quad ; \quad X^{i>3} = 0. \quad (23)$$

The worldvolume gauge field is taken to have only magnetic components:

$$\frac{1}{2} F_{\mu\nu} d\xi^\mu \wedge d\xi^\nu = F_{12} d\xi^1 \wedge d\xi^2. \quad (24)$$

Inserting these fields into (22) yields for the probe action,

$$S_2 = -\frac{T_0}{2\pi} \int d^3\xi \, h^{-1} \{ \sqrt{F_{12}^2 + h} - F_{12} \}. \quad (25)$$

The harmonic function  $h$  is given in (9), where  $r$  represents the spacetime separation between the D0-brane and a point on the (2+0) worldvolume. So,

$$h = N_2 Q_0 [(\xi^1)^2 + (\xi^2)^2 + (b_3)^2]^{-7/2}. \quad (26)$$

Now we turn to Matrix Theory to see if our ansatz can reproduce (25). In MT we represent we represent a (2+0) state lying in the (12) plane by

$$\begin{aligned} \mathcal{U}_1 &= \frac{Q}{\sqrt{F_{12}}} \quad ; \quad \mathcal{U}_2 = \frac{P}{\sqrt{F_{12}}} \\ \mathcal{U}_3 &= b_3 \quad ; \quad \mathcal{U}_{i>3} = 0 \quad ; \quad \mathcal{V}_i = \mathbf{0}_{N_2 \times N_2} . \end{aligned}$$

The MT field strength, as defined in (1),(2), is then

$$\frac{1}{2} \mathcal{F}_{MN} dX^M \wedge dX^N = -i \frac{[Q, P]}{F_{12}} dX^1 \wedge dX^2 = \frac{1}{F_{12}} dX^1 \wedge dX^2 \quad (27)$$

The “harmonic function”  $\mathfrak{h}$ , defined by (3), becomes

$$\mathfrak{h} = Q_0 \left( \frac{Q^2}{F_{12}} + \frac{P^2}{F_{12}} + b_3^2 \right)^{-7/2} \quad (28)$$

Working out the determinant in  $S_{\text{MT}}$  we find

$$\det[\eta_{MN} - \mathfrak{h}^{1/2} \mathcal{F}_{MN}] = - \left( 1 + \frac{\mathfrak{h}}{F_{12}^2} \right). \quad (29)$$

It remains to compute the traces.  $P, Q$  become the worldvolume coordinates when we convert the trace to an integral using (6) and rescale,

$$\frac{P}{\sqrt{F_{12}}} = \xi^1 ; \quad \frac{Q}{\sqrt{F_{12}}} = -\xi^2 \Rightarrow \text{Tr} = \frac{1}{2\pi} \int dQ dP = \frac{F_{12}}{2\pi} \int d^2\xi$$

Note that the integration measure is implicitly a two-form, which explains the positive sign for the  $d^2\xi$  measure.  $\text{Tr}^{(N_2)}$  simply acts by multiply  $\mathbf{h}$  by  $N_2$ . Putting the pieces together and substituting into (17) yields for the effective action

$$S_{MT} = -\frac{T_0}{2\pi} \int d^3\xi \, h^{-1} \{ \sqrt{F_{12}^2 + h} - F_{12} \}, \quad (30)$$

where

$$h = N_2 Q_0 [(\xi^1)^2 + (\xi^2)^2 + (b_3)^2]^{-7/2} \quad (31)$$

as in (26). The result indeed agrees with (25).

## 4.2 Rotated case

As our next check, let's consider the effect of rotating the D2-brane in the (13)-plane. Since the D0-brane source background is spherically symmetric, this rotation is rather trivial from a physical point of view; however, it constitutes a non-trivial check of our ansatz and illustrates the general procedure for rotating branes in more elaborate examples. It will also illustrate a relation between different gauge choices in supergravity actions, and the various ways of writing backgrounds in Matrix Theory.

In supergravity, with the usual static gauge, the rotated (2+0)-brane background is given by

$$X^{0,1,2} = \xi^{0,1,2} ; \quad X^3 = b_3 + \xi^1 \tan \theta . \quad (32)$$

Its action in the D0-background is

$$S_2 = -\frac{T_0}{2\pi} \int d^3\xi \, h^{-1} \{ \sqrt{h(1 + \tan^2 \theta) + F_{12}^2} - F_{12} \} , \quad (33)$$

where the harmonic function is

$$h = N_2 Q_0 [(\xi^1)^2 + (\xi^2)^2 + (b_3 + \xi^1 \tan \theta)^2]^{-7/2} . \quad (34)$$

The action looks simpler in an alternative gauge, where the spatial worldvolume coordinates measure the physical distance along the brane. This gauge choice is easily written with the help of rotated spacetime coordinates  $\bar{X}^M$  which are aligned with the brane:

$$\bar{X}^M = X^M \quad M \neq 1, 3 \quad (35)$$

$$\begin{pmatrix} \bar{X}^1 \\ \bar{X}^3 - b_3 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} X^1 \\ X^3 - b_3 \end{pmatrix} . \quad (36)$$

We fix the new gauge, which we will call the “aligned gauge”, by setting

$$\bar{X}^{0,1,2} = \bar{\xi}^{0,1,2}. \quad (37)$$

In these coordinates (2+0)-brane is located at  $\bar{X}^3 = b^3$ . The new worldvolume coordinates are related to the old ones by

$$\bar{\xi}^0 = \xi^0 \quad ; \quad \bar{\xi}^1 \cos \theta = \xi^1 \quad ; \quad \bar{\xi}^2 = \xi^2. \quad (38)$$

Therefore, the new worldvolume magnetic field  $F_{\bar{1}\bar{2}}$  is related to  $F_{12}$  by

$$F_{\bar{1}\bar{2}} = \cos \theta F_{12}. \quad (39)$$

The D2-brane action is now

$$S_2 = -\frac{T_0}{2\pi} \int d^3 \bar{\xi} \bar{h}^{-1} \{ \sqrt{F_{\bar{1}\bar{2}}^2 + \bar{h}} - F_{\bar{1}\bar{2}} \}, \quad (40)$$

where the harmonic function is simply

$$\bar{h} = N_2 Q_0 [(\bar{\xi}^1)^2 + (\bar{\xi}^2)^2 + (b_3)^2]^{-7/2}. \quad (41)$$

Naturally, the two forms of the action (33),(40) are related by the relations of the barred and unbarred variables.

In the rotated frame, it is simple to write down the MT background:

$$\begin{aligned} \mathcal{U}_{\bar{1}} &= \frac{Q}{\sqrt{F_{\bar{1}\bar{2}}}} \quad ; \quad \mathcal{U}_2 = \frac{P}{\sqrt{F_{\bar{1}\bar{2}}}} \\ \mathcal{U}_{\bar{3}} &= b_3 \quad ; \quad \mathcal{U}_{i>3} = 0 \quad ; \quad \mathcal{V}_i = \mathbf{0}_{N_2 \times N_2}. \end{aligned}$$

The MT effective action reduces to the supergravity action for the probe in the aligned gauge, and this will continue to hold true as we return back to the static gauge. To make contact with the static gauge supergravity action, we rewrite the MT background by rotating back to the original unbarred coordinates. To do this correctly, we note that  $\mathcal{U}_{\bar{1}}, \mathcal{U}_{\bar{2}}$  are related to the spacetime coordinates  $\bar{X}^2, \bar{X}^1$ :

$$-\mathcal{U}_{\bar{1}} = \bar{\xi}^2 = \bar{X}^2 \quad ; \quad \mathcal{U}_2 = \bar{\xi}^1 = \bar{X}^1.$$

Therefore, a spacetime rotation in the (13) plane becomes a rotation in the (23)-plane of the  $\mathcal{U}$ 's:

$$\begin{pmatrix} -\mathcal{U}_{\bar{1}} \\ \mathcal{U}_2 \\ \mathcal{U}_{\bar{3}} - b_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} -\mathcal{U}_{\bar{1}} \\ \mathcal{U}_{\bar{2}} \\ \mathcal{U}_{\bar{3}} - b_3 \end{pmatrix}. \quad (42)$$

So, the background given in terms of  $\mathcal{U}$ 's is

$$\mathcal{U}_1 = \mathcal{U}_{\bar{1}} = \sqrt{\frac{1}{\cos \theta F_{12}}} Q \quad (43)$$

$$\mathcal{U}_2 = \cos \theta \mathcal{U}_{\bar{2}} = \sqrt{\frac{\cos \theta}{F_{12}}} P \quad (44)$$

$$\mathcal{U}_3 = b_3 + \sin \theta \mathcal{U}_{\bar{2}} = b_3 + \tan \theta \sqrt{\frac{\cos \theta}{F_{12}}} P . \quad (45)$$

Thus, the MT field strength tensor is

$$\mathcal{F}_{MN} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ & 0 & F_{12}^{-1} & \tan \theta F_{12}^{-1} & \\ \ddots & & 0 & 0 & \\ & \ddots & & 0 & \\ & & & & \ddots \end{pmatrix} \quad (46)$$

and the harmonic function is

$$h = Q_0 \left[ \frac{Q^2}{\cos \theta F_{12}} + \frac{\cos \theta}{F_{12}} P^2 + (b_3 + \tan \theta \sqrt{\frac{\cos \theta}{F_{12}}} P)^2 \right]^{-7/2} . \quad (47)$$

Changing integration variables:

$$\sqrt{\frac{\cos \theta}{F_{12}}} P = \xi^1 ; \quad \sqrt{\frac{1}{\cos \theta F_{12}}} Q = -\xi^2 \Rightarrow \text{Tr} = \frac{1}{2\pi} \int dQ dP = \frac{F_{12}}{2\pi} \int d^2 \xi . \quad (48)$$

The harmonic function now takes the same form (34) as in the supergravity calculation. Also, inserting the field strength (46) into the MT effective action (17), we recover the supergravity action (33) in static gauge.

The preceeding discussion was quite detailed, but relations like the ones above are needed when one moves on to consider the more complicated examples of non-extremal, boosted, and rotated brane configurations.

### 4.3 Two rotations

As our next example, we add another rotation. We take the D2-brane to be rotated by the angle  $\theta_1$  in (13)-plane and by the angle  $\theta_2$  in (24)-plane. The two coordinate systems are related by

$$\begin{pmatrix} \bar{X}^1 \\ \bar{X}^2 \\ \bar{X}^3 - b_3 \\ \bar{X}^4 - b_4 \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & 0 & \sin \theta_1 & 0 \\ 0 & \cos \theta_2 & 0 & \sin \theta_2 \\ -\sin \theta_1 & 0 & \cos \theta_1 & 0 \\ 0 & -\sin \theta_2 & 0 & \cos \theta_2 \end{pmatrix} \begin{pmatrix} X^1 \\ X^2 \\ X^3 - b_3 \\ X^4 - b_4 \end{pmatrix} . \quad (49)$$

The two ways to describe the brane are then

$$X^{1,2} = \xi^{1,2} ; \quad X^3 = \tan \theta_1 \xi^1 + b_3 ; \quad X^4 = \tan \theta_2 \xi^2 + b_4 \quad (\text{static}) \quad (50)$$

$$\bar{X}^{1,2} = \bar{\xi}^{1,2} ; \quad \bar{X}^3 = b_3 ; \quad \bar{X}^4 = b_4 \quad (\text{aligned}) . \quad (51)$$

The supergravity probe action is

$$S = -\frac{T_0}{2\pi} \int d^3 \bar{\xi} \bar{h}^{-1} \{ \sqrt{\bar{h} + F_{12}^2} - F_{12} \} \quad (\text{aligned}) \quad (52)$$

$$= -\frac{T_0}{2\pi} \int d^3 \xi h^{-1} \{ \sqrt{h(1 + \tan^2 \theta_1)(1 + \tan^2 \theta_2) + F_{12}^2} - F_{12} \} \quad (\text{static}) \quad (53)$$

where the harmonic function is

$$\bar{h} = N_2 Q_0 [(\bar{\xi}^1)^2 + (\bar{\xi}^2)^2 + (b_3)^2 + (b_4)^2]^{-7/2} \quad (\text{aligned}) \quad (54)$$

$$h = N_2 Q_0 [(\xi^1)^2 + (\xi^2)^2 + (b_3 + \xi^1 \tan \theta_1)^2 + (b_4 + \xi^2 \tan \theta_2)^2]^{-7/2} \quad (\text{static}). \quad (55)$$

In the barred variables, the MT background is

$$\mathcal{U}_{\bar{1}} = \frac{Q}{\sqrt{F_{12}}} ; \quad \mathcal{U}_{\bar{2}} = \frac{P}{\sqrt{F_{12}}} ; \quad \mathcal{U}_{\bar{3},\bar{4}} = b_{3,4} . \quad (56)$$

The worldvolume coordinates and magnetic fields are related by

$$\xi^0 = \bar{\xi}^0 ; \quad \xi^{1,2} = \bar{\xi}^{1,2} \cos \theta_{1,2} ; \quad F_{12} = \cos \theta_1 \cos \theta_2 F_{12} . \quad (57)$$

Rotating to the original frame, the (13) and (24) rotations correspond to the (23) and (14) rotations for the  $\mathcal{U}$ 's :

$$\begin{pmatrix} -\mathcal{U}_1 \\ \mathcal{U}_2 \\ \mathcal{U}_3 - b_3 \\ \mathcal{U}_4 - b_4 \end{pmatrix} = \begin{pmatrix} \cos \theta_2 & 0 & 0 & -\sin \theta_2 \\ 0 & \cos \theta_1 & -\sin \theta_1 & 0 \\ 0 & \sin \theta_1 & \cos \theta_1 & 0 \\ \sin \theta_2 & 0 & 0 & \cos \theta_2 \end{pmatrix} \begin{pmatrix} -\mathcal{U}_{\bar{1}} \\ \mathcal{U}_{\bar{2}} \\ \mathcal{U}_{\bar{3}} - b_3 \\ \mathcal{U}_{\bar{4}} - b_4 \end{pmatrix} . \quad (58)$$

This yields

$$\mathcal{U}_1 = \cos \theta_2 \mathcal{U}_{\bar{1}} = \sqrt{\frac{\cos \theta_2}{\cos \theta_1 F_{12}}} Q \quad (59)$$

$$\mathcal{U}_2 = \cos \theta_1 \mathcal{U}_{\bar{2}} = \sqrt{\frac{\cos \theta_1}{\cos \theta_2 F_{12}}} P \quad (60)$$

$$\mathcal{U}_3 = b_3 + \sin \theta_1 \mathcal{U}_{\bar{2}} = b_3 + \tan \theta_1 \mathcal{U}_2 \quad (61)$$

$$\mathcal{U}_4 = b_4 - \sin \theta_2 \mathcal{U}_{\bar{1}} = b_4 - \tan \theta_2 \mathcal{U}_1 , \quad (62)$$

so the MT field strength  $\mathcal{F}_{MN}$  is related to the probe field strength  $F_{\mu\nu}$  by

$$\mathcal{F}_{MN} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ & 0 & F_{12}^{-1} & \tan \theta_1 F_{12}^{-1} & 0 & 0 & \\ \ddots & & 0 & 0 & \tan \theta_2 F_{12}^{-1} & 0 & \\ & \ddots & & 0 & -\tan \theta_1 \tan \theta_2 F_{12}^{-1} & 0 & \\ & & \ddots & & 0 & 0 & \\ & & & & & \ddots & \end{pmatrix}. \quad (63)$$

Notice the component  $\mathcal{F}_{34}$ , which has a quadratic dependence on the slopes; it appears since  $\mathcal{U}_3, \mathcal{U}_4$  above do not commute.

The harmonic function  $\mathbf{h}$  in the MT effective action is

$$\mathbf{h} = N_2 Q_0 \{ \mathcal{U}_1^2 + \mathcal{U}_2^2 + (b_3 + \tan \theta_1 \mathcal{U}_2)^2 + (b_4 - \tan \theta_2 \mathcal{U}_1)^2 \}^{-7/2}. \quad (64)$$

With the change of variables

$$\mathcal{U}_2 = \sqrt{\frac{\cos \theta_1}{\cos \theta_2 F_{12}}} P = \xi^1; \quad \mathcal{U}_1 = \sqrt{\frac{\cos \theta_2}{\cos \theta_1 F_{12}}} Q = -\xi^2 \Rightarrow \text{Tr} = \frac{1}{2\pi} \int dQ dP = \frac{F_{12}}{2\pi} \int d^2 \xi \quad (65)$$

$\mathbf{h}$  becomes

$$\mathbf{h} = N_2 Q_0 \{ (\xi^1)^2 + (\xi^2)^2 + (b_3 + \xi^1 \tan \theta_1)^2 + (b_4 + \xi^2 \tan \theta_2)^2 \}^{-7/2}, \quad (66)$$

in agreement with the harmonic function  $h$  in supergravity (55). Substituting in  $\mathbf{h}$  (66) and  $\mathcal{F}_{MN}$  (63), the MT effective action (17) reduces to the supergravity action (53) for the probe in the static gauge.

We now move on to more general probes, and add electric fields and transverse velocities to the backgrounds.

## 5 $0_s - (p + (p - 2) + \cdots + 0)_p$ brane interactions

In this section, we consider  $p + (p - 2) + \cdots + 0$  brane probes (with  $p = 2n$ ) in the 0-brane background. The results we find will generalize those of the previous section and will highlight the remarkable way in which Matrix Theory can accomodate a wide variety of branes. To check our ansatz in a more detailed manner than before, we will turn on the electric components  $F_{0i}$  of the worldvolume gauge field  $F_{\mu\nu}$  in addition to the magnetic components. We will show that the MT effective action ansatz (17) reproduces the supergravity action for the general p-brane probe. Finally, we return to the example of

the (2+0)-brane probe, adding both transverse velocities and rotations, and demonstrate the resulting agreement with supergravity.

Once again, we start with the supergravity action for the probe,

$$S_p = -T_p \int d^{p+1} \xi \left\{ h^{-3/4} \sqrt{\det[g_{\mu\nu} - F_{\mu\nu}] } - h^{-1} \text{Pf}(-B) \right\} . \quad (67)$$

where we use the notation  $B_{ij}$  for the spatial part of the worldvolume field strength of the  $2n + 2(n-1) + \dots + 0$  brane probe:

$$(B_{ij}) = (F_{ij}) = \begin{pmatrix} 0 & F_{12} & & & \\ -F_{12} & 0 & & & \\ & & 0 & F_{34} & \\ & & -F_{34} & 0 & \\ & & & & \ddots \end{pmatrix} , \quad (68)$$

and we have made use of the Pfaffian:

$$\text{Pf}(B) = \frac{(-1)^n}{2^n n!} \epsilon_{i_1 \dots i_{2n}} B_{i_1 i_2} \dots B_{i_{2n-1} i_{2n}} . \quad (69)$$

In the square root term of the action, there appears determinant of the matrix

$$g - F = \begin{pmatrix} h^{-1/2}(1 - hv^2) & -\mathbf{E}^T \\ \mathbf{E} & -h^{1/2}\mathbf{1} - B \end{pmatrix} , \quad (70)$$

where we use the notation

$$\mathbf{E} = (E_i) = (F_{0i}) \quad (71)$$

for the electric field components. We have also included a transverse velocity  $v$  in the direction  $X^{p+1}$ . Next, we manipulate the square root of the determinant by inserting a factor

$$1 = \sqrt{\det \begin{pmatrix} 1 & \\ & -B \end{pmatrix} \det \begin{pmatrix} 1 & \\ & -B^{-1} \end{pmatrix}} = \text{Pf}(-B) \sqrt{\det \begin{pmatrix} 1 & \\ & -B^{-1} \end{pmatrix}} \quad (72)$$

and pulling out a factor  $h^{-1/4}$ . Then the action (67) takes the form

$$S_p = -T_p \int d^{p+1} \xi \text{Pf}(-B) h^{-1} \{ \sqrt{\det \mathbf{M}_1} - 1 \} , \quad (73)$$

where

$$\det \mathbf{M}_1 \equiv \det \begin{pmatrix} 1 - hv^2 & -h^{1/2} \mathbf{E}^T \\ -B^{-1} \mathbf{E} & \mathbf{1} + h^{1/2} B^{-1} \end{pmatrix} . \quad (74)$$

We will now compare the above form of the supergravity action with the MT effective action ansatz

$$S_{MT} = -T_0 \text{Tr}^{(N_1)} \int dt \, \mathbf{h}^{-1} \left\{ \sqrt{-\det[\eta - \mathbf{h}^{1/2} \mathcal{F}] - 1} \right\} . \quad (75)$$

The  $\text{Tr}^{(N_2)}$  operation has already been performed, and its effect has been taken into account by including a factor  $N_2$  into the “harmonic function”  $\mathbf{h}$ . Next, we map the action (75) to an action on the  $p+1$ -dimensional worldvolume, using the rule

$$T_0 \text{Tr}^{(N)} \int dt \rightarrow T_p \int d^{p+1} \xi \, F_{12} F_{34} \cdots = T_p \int d^{p+1} \xi \, \text{Pf}(-B) , \quad (76)$$

and relating the matrix field strength  $\mathcal{F}$  to the  $p$ -brane worldvolume field strength  $F$  and the transverse velocity  $v$  by the identities

$$\mathcal{F} = \hat{g} \hat{F} \hat{g} \quad (77)$$

$$\hat{g} = \text{diag}(1, \underbrace{-F_{12}^{-1}, -F_{12}^{-1}, -F_{34}^{-1}, -F_{34}^{-1}, \dots}_p, \underbrace{-1, \dots, -1}_{9-p}) \quad (78)$$

$$\hat{F} = \begin{pmatrix} 0 & \overbrace{F_{0j}}^p & v & 0 & \cdots \\ -F_{0i} & F_{ij} & 0 & & \\ -v & 0 & 0 & & \\ 0 & & & \ddots & \\ \vdots & & & & \end{pmatrix} . \quad (79)$$

These relations are the same as those used in previous sections, but written in a different notation. The harmonic function  $\mathbf{h}$  is reduced to the supergravity form  $h$  as before. With these substitutions, the MT effective action takes the form

$$S_{MT} = -T_p \int d^{p+1} \xi \, \text{Pf}(-B) h^{-1} \left\{ \sqrt{-\det[\eta - h^{1/2} \hat{g} \hat{F} \hat{g}] - 1} \right\} . \quad (80)$$

Obviously, this is equal to the supergravity action (73), iff the determinant terms under the square roots agree. To show this, we first rewrite the determinant term in (80) as follows:

$$-\det[\eta - h^{1/2} \hat{g} \hat{F} \hat{g}] = \det[\mathbf{1} - h^{1/2} \eta \hat{g}^2 \hat{F}] . \quad (81)$$

Substituting<sup>2</sup>

$$\eta \hat{g}^2 = \text{diag}(1, -F_{12}^{-2}, -F_{12}^{-2}, \dots, -1, -1, \dots, -1) = \begin{pmatrix} 1 & & \\ & B^{-2} & \\ & & -\mathbf{1} \end{pmatrix} \quad (82)$$

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<sup>2</sup>Note that  $\eta \hat{g}^2$  is equal to the metric  $\hat{\eta}$ , the rescaling of the spatial coordinates, which was discussed in Refs. [14, 21].



and using the expression (79) for  $\hat{F}$ , we get

$$-\det[\eta - h^{1/2}\hat{g}\hat{F}\hat{g}] = \det \begin{pmatrix} 1 & -h^{1/2}\mathbf{E}^T & -h^{1/2}v & 0 \\ h^{1/2}B^{-2}\mathbf{E} & \mathbf{1} - h^{1/2}B^{-1} & 0 & \\ -h^{1/2}v & 0 & 1 & \\ 0 & & & \mathbf{1} \end{pmatrix}. \quad (83)$$

The  $10 \times 10$  determinant above is equal to the  $(p+1) \times (p+1)$  determinant

$$\det \begin{pmatrix} 1 - hv^2 & -h^{1/2}\mathbf{E}^T \\ h^{1/2}B^{-2}\mathbf{E} & \mathbf{1} - h^{1/2}B^{-1} \end{pmatrix} \equiv \det \mathbf{M}_2. \quad (84)$$

Then, finally, the agreement of the MT effective action and the supergravity effective action is implied by the following determinant identity

$$\det \mathbf{M}_2 = \det \mathbf{M}_1, \quad (85)$$

which we have verified by an explicit evaluation of the two determinants.

To conclude this section, we consider an example which combines the results given above with the results of the previous section: we consider a 2+0 brane with electric fields, rotated in the (13) and (23) planes, and with a velocity in direction 3. In the static gauge, the background is given by

$$X^{0,1,2} = \xi^{0,1,2}; \quad X^3 = b_3 + vt + \xi^1 \tan \theta_1 + \xi^2 \tan \theta_2. \quad (86)$$

The  $(2+0)_p$  action is

$$S_2 = -T_2 \int d^3\xi \left\{ h^{-3/4} \sqrt{\det[g_{\mu\nu} - F_{\mu\nu}]} - h^{-1} F_{12} \right\}, \quad (87)$$

where

$$(g_{\mu\nu} - F_{\mu\nu}) = \begin{pmatrix} h^{1/2}(1 - hv^2) & -h^{1/2}v \tan \theta_1 - F_{01} & -h^{1/2}v \tan \theta_2 - F_{02} \\ \ddots & -h^{1/2}(1 + \tan^2 \theta_1) & -h^{1/2} \tan \theta_1 \tan \theta_2 - F_{12} \\ & & -h^{1/2}(1 + \tan^2 \theta_2) \end{pmatrix} \quad (88)$$

and the harmonic function is

$$h = N_1 Q_0 \{(\xi^1)^2 + (\xi^2)^2 + (b_3 + \xi^1 \tan \theta_1 + \xi^2 \tan \theta_2)^2\}^{-7/2}. \quad (89)$$

To find the MT background  $\mathcal{U}_i$  and the MT field strength, we proceed as before: we first go to the aligned gauge, thus eliminating the rotation contribution. Then we write down the MT background in the rotated frame (recalling that the origin of the coordinates is

set to the brane, as in the previous section), and rotate back to the original frame. The result is as follows:

$$\begin{aligned}
\mathcal{U}_1 &= \sqrt{\mathcal{F}_{12}} Q + \mathcal{F}_{01} t \\
\mathcal{U}_2 &= \sqrt{\mathcal{F}_{12}} P + \mathcal{F}_{02} t \\
\mathcal{U}_3 &= \frac{\mathcal{F}_{13}}{\sqrt{\mathcal{F}_{12}}} P - \frac{\mathcal{F}_{23}}{\sqrt{\mathcal{F}_{12}}} Q + b_3 + \mathcal{F}_{03} t ,
\end{aligned} \tag{90}$$

with the MT field strength  $\mathcal{F}_{MN}$  given by

$$\mathcal{F}_{MN} = \begin{pmatrix} 0 & -F_{01}F_{12}^{-1} & -F_{02}F_{12}^{-1} & -(\tan \theta_1 F_{02} - \tan \theta_2 F_{01})F_{12}^{-1} + v & 0 & \cdots \\ \ddots & 0 & F_{12}^{-1} & \tan \theta_1 F_{12}^{-1} & 0 & \\ & \ddots & 0 & \tan \theta_2 F_{12}^{-1} & 0 & \\ & & \ddots & 0 & 0 & \\ & & & \ddots & & \ddots \end{pmatrix} . \tag{91}$$

The MT harmonic function is

$$\mathfrak{h} = N_2 Q_0 \{ \mathcal{U}_1^2 + \mathcal{U}_2^2 + \mathcal{U}_3^2 \}^{-7/2} . \tag{92}$$

With the change of variables

$$\mathcal{U}_2 = \xi^1 ; \mathcal{U}_1 = -\xi^2 \Rightarrow \text{Tr} = \frac{1}{2\pi} \int dQ dP = \frac{F_{12}}{2\pi} \int d^2 \xi \tag{93}$$

the harmonic function becomes

$$\mathfrak{h} = N_2 Q_0 \{ (\xi^1)^2 + (\xi^2)^2 + (b_3 + \xi^1 \tan \theta_1 + \xi^2 \tan \theta_2)^2 \}^{-7/2} , \tag{94}$$

in agreement with the form (89). Further, substituting  $\mathcal{F}_{MN}$  from above into the MT effective action (17), we correctly reproduce the supergravity probe action (87). Thus, even this complicated check for non-extremal, boosted and rotated branes passes the test.

## 6 D4-branes as sources

We have seen that our ansatz correctly reproduces a wide variety of actions for probes moving in the background of a D0-brane source. We now replace the D0-brane source by a bound state of D4-branes and D0-branes: the (4+0) configuration. This replacement adds a great deal of additional structure to the probe actions and serves as a highly non-trivial check of our methods. However, for reasons discussed in section 3 we will confine our

attention to one loop results; it is an important challenge to resolve the trace ambiguities that would allow for checks to be made for higher loops.

In supergravity the (4+0) configuration is described by the fields [36]

$$\begin{aligned} ds^2 &= h^{-1/2} H_4^{-1/2} dt^2 - h^{1/2} H_4^{-1/2} (dx_1^2 + \cdots + dx_4^2) - h^{1/2} H_4^{1/2} (dx_5^2 + \cdots + dx_9^2) \\ e^{-\phi} &= h^{-3/4} H_4^{1/4} \quad ; \quad C_t^{(1)} = h^{-1} \quad ; \quad C_{t1234}^{(5)} = H_4^{-1} - 1 \\ h &= \frac{\tilde{F}^2 N_4 Q_0^{(4)}}{(2\pi)^2 r_\perp^3} = \frac{\tilde{F}^2 N_4 Q_0}{15 r_\perp^3} \quad ; \quad H_4 = 1 + N_4 h_4 = 1 + \frac{N_4 Q_4}{r_\perp^3} = 1 + \frac{N_4 Q_0}{15 r_\perp^3} . \end{aligned}$$

We are following the notation in [17]:  $Q_0^{(4)}$  is the charge density of a D0-brane “smeared” in four directions,  $Q_4$  is the D4-brane charge, and  $N_4$  is the number of D4-branes. The Matrix theory background for the (4+0) state is obtained by combining two (4+2+2+0) states in such a way as to cancel off the D2-brane charge. Explicitly [31]:

$$\begin{aligned} \mathcal{V}_1 &= \frac{1}{\sqrt{\tilde{F}}} \begin{pmatrix} \tilde{Q}_1 & 0 \\ 0 & \tilde{Q}_1 \end{pmatrix} \quad ; \quad \mathcal{V}_2 = \frac{1}{\sqrt{\tilde{F}}} \begin{pmatrix} \tilde{P}_1 & 0 \\ 0 & -\tilde{P}_1 \end{pmatrix} \\ \mathcal{V}_3 &= \frac{1}{\sqrt{\tilde{F}}} \begin{pmatrix} \tilde{Q}_2 & 0 \\ 0 & \tilde{Q}_2 \end{pmatrix} \quad ; \quad \mathcal{V}_4 = \frac{1}{\sqrt{\tilde{F}}} \begin{pmatrix} \tilde{P}_2 & 0 \\ 0 & -\tilde{P}_2 \end{pmatrix} \\ \mathcal{V}_{i>3} &= 0. \end{aligned} \tag{95}$$

The property  $\text{Tr} [\mathcal{V}_i, \mathcal{V}_j] = 0$  implies the absence of D2-branes. The configuration represents two D4-branes bound to a total density  $\sigma_0 = 2(\tilde{F}/2\pi)^2$  of D0-branes.

## 6.1 D0 probe

Let us first take the probe to be a D0-brane. This case has been considered before in [17] using different techniques, but we present it here in order to illustrate our methods in a simple context.

From supergravity,

$$\begin{aligned} S_0 &= -N_1 T_0 \int dt \{ e^{-\phi} \sqrt{g_{MN} \dot{X}^N \dot{X}^M} - C_t^{(1)} \} \\ &= -N_1 T_0 \int dt \, h^{-1} \{ \sqrt{1 - h v_\parallel^2 - h H_4 v_\perp^2} - 1 \} \\ &= \mathcal{O}(h_4^0) - \frac{N_1 N_4}{8} [\tilde{F}^2 (v_\parallel^2 + v_\perp^2)^2 + 4 v_\perp^2] h_4 + \mathcal{O}(h_4^2). \end{aligned} \tag{96}$$

Here  $v_\parallel^2 = v_1^2 + \cdots + v_4^2$ ,  $v_\perp^2 = v_5^2 + \cdots + v_9^2$ , and we keep only the one loop contribution.

From Matrix theory

$$\mathcal{U}_i = (b_i + v_i t) \otimes \mathbf{1}_{2 \times 2} \tag{97}$$

so,

$$\frac{1}{2}\mathcal{F}_{MN}dX^M\wedge dX^N = (v_i\mathbf{1}_{N_1\times N_1}\otimes\mathbf{1}_{2\times 2})(dX^0\wedge dX^i) + (\frac{1}{\tilde{F}}\mathbf{1}_{N_1\times N_1}\otimes\sigma_3)(dX^1\wedge dX^2 + dX^3\wedge dX^4), \quad (98)$$

and

$$\mathbf{h} = Q_0 [\mathcal{U}_i \otimes \mathbf{1}_{2\times 2} - \mathbf{1}_{N_1\times N_1} \otimes \mathcal{V}_i^*]^{-7} = Q_0 \mathbf{1}_{N_1\times N_1} \otimes \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}^{-7/2} \quad (99)$$

where

$$\begin{aligned} \alpha &= (b_1 + v_1 t - \frac{\tilde{Q}_1}{\sqrt{\tilde{F}}})^2 + (b_2 + v_2 t - \frac{\tilde{P}_1}{\sqrt{\tilde{F}}})^2 + (b_3 + v_3 t - \frac{\tilde{Q}_2}{\sqrt{\tilde{F}}})^2 + (b_4 + v_4 t - \frac{\tilde{P}_2}{\sqrt{\tilde{F}}})^2 \\ &\quad + (b_\perp + v_\perp t)^2 \\ \beta &= (b_1 + v_1 t - \frac{\tilde{Q}_1}{\sqrt{\tilde{F}}})^2 + (b_2 + v_2 t + \frac{\tilde{P}_1}{\sqrt{\tilde{F}}})^2 + (b_3 + v_3 t - \frac{\tilde{Q}_2}{\sqrt{\tilde{F}}})^2 + (b_4 + v_4 t + \frac{\tilde{P}_2}{\sqrt{\tilde{F}}})^2 \\ &\quad + (b_\perp + v_\perp t)^2. \end{aligned} \quad (100)$$

The Matrix theory action is

$$S_{\text{MT}} = \mathcal{O}(\mathbf{h}^0) - \frac{1}{8}T_0 \text{Tr}^{(N_1)}\text{Tr}^{(N_2)} \int dt \mathbf{h} \left[ \text{tr}(\eta\mathcal{F})^4 - \frac{1}{4}(\text{tr}(\eta\mathcal{F})^2)^2 \right] + \mathcal{O}(\mathbf{h}^2) .$$

Plugging in the background we find,

$$\text{tr}(\eta\mathcal{F})^4 - \frac{1}{4}(\text{tr}(\eta\mathcal{F})^2)^2 = \left[ (v_\parallel^2 + v_\perp^2)^2 + \frac{4v_\perp^2}{\tilde{F}^2} \right] \mathbf{1}_{N_1\times N_1} \otimes \mathbf{1}_{2\times 2}.$$

Now,

$$\text{Tr}^{(N_1)} \rightarrow N_1 \quad ; \quad \text{Tr}^{(N_2)} \rightarrow \frac{1}{(2\pi)^2} \int d\tilde{P}_1 d\tilde{Q}_1 d\tilde{P}_2 d\tilde{Q}_2 \text{Tr}^{(2\times 2)}.$$

Defining

$$(\tilde{\xi}^1, \tilde{\xi}^2, \tilde{\xi}^3, \tilde{\xi}^4) = \left( \frac{\tilde{P}_1}{\sqrt{\tilde{F}}}, \frac{\tilde{Q}_1}{\sqrt{\tilde{F}}}, \frac{\tilde{P}_2}{\sqrt{\tilde{F}}}, \frac{\tilde{Q}_2}{\sqrt{\tilde{F}}} \right)$$

we have

$$\text{Tr}^{(N_2)} \rightarrow \frac{\tilde{F}^2}{(2\pi)^2} \int d^4\tilde{\xi} \text{Tr}^{(2\times 2)}.$$

We also need

$$\frac{1}{(2\pi)^2} \int d^4\tilde{\xi} \mathbf{h} = \frac{Q_0}{(2\pi)^2} \int d^4\tilde{\xi} \left[ (\tilde{\xi}^1)^2 + \dots + (\tilde{\xi}^4)^2 + (b_\perp + v_\perp t)^2 \right]^{-7/2} \mathbf{1}_{2\times 2} \quad (101)$$

$$= \frac{Q_0}{15(b_\perp + v_\perp t)^3} \mathbf{1}_{2\times 2} = \frac{Q_4}{r_\perp^3} \mathbf{1}_{2\times 2} = h_4 \mathbf{1}_{2\times 2}. \quad (102)$$

Putting it all together we find

$$S_{\text{MT}} = \mathcal{O}(\mathbf{h}^0) - \frac{2N_1}{8} [\tilde{F}^2(v_\parallel^2 + v_\perp^2)^2 + 4v_\perp^2] h_4 + \mathcal{O}(\mathbf{h}^2) \quad (103)$$

in agreement with (96) when we set  $N_4 = 2$ .

## 6.2 D(4+0) probe

For our final example we will consider the interaction of branes oriented at a relative angle [37]. The source will again be the (4+0) configuration lying in the (1234) plane. For the probe, we start with a (4+2+2+0) configuration in the (1234) plane and then rotate in the (15) plane by an angle  $\theta$ .<sup>3</sup> We will choose the (4+2+2+0) state to be at the self-dual point, so that the force between the branes will vanish when  $\theta = 0$ .

We begin with supergravity. The (4+2+2+0) probe is described by

$$X^{0,1,2,3,4} = \xi^{0,1,2,3,4} \quad ; \quad X^5 = \xi^1 \tan \theta \quad ; \quad X^6 = b_6$$

$$\frac{1}{2} F_{\mu\nu} d\xi^\mu \wedge d\xi^\nu = F_{12} d\xi^1 \wedge d\xi^2 + F_{34} d\xi^3 \wedge d\xi^4.$$

The probe action is

$$\begin{aligned} S_4 &= -\frac{T_0}{(2\pi)^2} \int d^5 \xi \left\{ e^{-\phi} \sqrt{\det[g_{MN} \partial_\mu X^M \partial_\nu X^N - F_{\mu\nu}]} - F_{12}^2 C_t^{(1)} - C_{t1234}^{(5)} \right\} \\ &= -\frac{T_0}{(2\pi)^2} \int d^5 \xi \left\{ h^{-1} [\sqrt{(hH_4^{-1} + F_{12}^2)(hH_4^{-1} + F_{12}^2 + h \tan \theta)} - F_{12}^2] - H_4^{-1} + 1 \right\} \\ &= \mathcal{O}(h_4^0) - \frac{T_0}{4(2\pi)^2} \int d^5 \xi \frac{\tilde{F}^2}{F_{12}^2} \tan^4 \theta h_4 + \mathcal{O}(h_4^2) \end{aligned} \quad (104)$$

where

$$h_4 = \frac{Q_0}{15[b_6^2 + (\xi^1)^2 \tan^2 \theta]^{3/2}}.$$

Now we consider Matrix theory. The (4+2+2+0) probe background is

$$\begin{aligned} \mathcal{U}_1 &= \frac{Q_1}{\sqrt{F_{12} \cos \theta}} \quad ; \quad \mathcal{U}_2 = \frac{\sqrt{\cos \theta} P_1}{\sqrt{F_{12}}} \\ \mathcal{U}_3 &= \frac{Q_2}{\sqrt{F_{12}}} \quad ; \quad \mathcal{U}_4 = \frac{P_2}{\sqrt{F_{12}}} \\ \mathcal{U}_5 &= \frac{\tan \theta P_1}{\sqrt{F_{12} \cos \theta}} \quad ; \quad \mathcal{U}_6 = b_6, \end{aligned} \quad (105)$$

where we have included the  $\theta$  dependence following the method described in section 4. Using the  $\mathcal{V}_i$  as in (95) we compute  $\mathcal{F}_{MN}$ :

$$\frac{1}{2} \mathcal{F}_{MN} dX^M \wedge dX^N = \left( \frac{1}{F_{12}} \mathbf{1}_{2 \times 2} - \frac{1}{\tilde{F}} \sigma_3 \right) (dX^1 \wedge dX^2 + dX^3 \wedge dX^4) + \frac{\tan \theta}{F_{12}} \mathbf{1}_{2 \times 2} dX^1 \wedge dX^5, \quad (106)$$

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<sup>3</sup>A different configuration of D4-branes at angles was recently studied in [39].

and

$$\mathbf{h} = Q_0[\mathcal{U}_i \otimes \mathbf{1}_{2 \times 2} - \mathcal{V}_i^*]^{-7} = Q_0 \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}^{-7/2} \quad (107)$$

with

$$\begin{aligned} \alpha &= \left( \frac{Q_1}{\sqrt{F_{12}}} - \frac{\tilde{Q}_1}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{P_1}{\sqrt{F_{12}}} - \frac{\tilde{P}_1}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{Q_2}{\sqrt{F_{12}}} - \frac{\tilde{Q}_2}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{P_2}{\sqrt{F_{12}}} - \frac{\tilde{P}_2}{\sqrt{\tilde{F}}} \right)^2 + b_6^2 \\ \beta &= \left( \frac{Q_1}{\sqrt{F_{12}}} - \frac{\tilde{Q}_1}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{P_1}{\sqrt{F_{12}}} + \frac{\tilde{P}_1}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{Q_2}{\sqrt{F_{12}}} - \frac{\tilde{Q}_2}{\sqrt{\tilde{F}}} \right)^2 + \left( \frac{P_2}{\sqrt{F_{12}}} + \frac{\tilde{P}_2}{\sqrt{\tilde{F}}} \right)^2 + b_6^2. \end{aligned}$$

We now find

$$\text{tr}(\eta\mathcal{F})^4 - \frac{1}{4}(\text{tr}(\eta\mathcal{F})^2)^2 = \frac{\tan^4\theta}{F_{12}^4} \mathbf{1}_{2 \times 2}. \quad (108)$$

In a similar manner to the D0-brane probe example, we have

$$\begin{aligned} \text{Tr}^{(N_1)} &\rightarrow \frac{1}{(2\pi)^2} \int dP_1 dQ_1 dP_2 dQ_2 \text{Tr}^{(2 \times 2)} = \frac{F_{12}^2}{(2\pi)^2} \int d^4\xi \text{Tr}^{(2 \times 2)} \\ \text{Tr}^{(N_2)} &\rightarrow \frac{1}{(2\pi)^2} \int d\tilde{P}_1 d\tilde{Q}_1 d\tilde{P}_2 d\tilde{Q}_2 \text{Tr}^{(2 \times 2)} = \frac{\tilde{F}^2}{(2\pi)^2} \int d^4\tilde{\xi} \text{Tr}^{(2 \times 2)} \end{aligned} \quad (109)$$

and

$$\frac{1}{(2\pi)^2} \int d^4\tilde{\xi} \mathbf{h} = \frac{Q_0}{15[b_6^2 + (\xi^1)^2 \tan^2\theta]^{3/2}} \mathbf{1}_{2 \times 2} = h_4 \mathbf{1}_{2 \times 2}. \quad (110)$$

Plugging into  $S_{\text{MT}}$  we find,

$$S_{\text{MT}} = \mathcal{O}(\mathbf{h}^0) - \frac{T_0}{4(2\pi)^2} \int d^5\xi \frac{\tilde{F}^2}{F_{12}^2} \tan^4\theta h_4 + \mathcal{O}(\mathbf{h}^2) \quad (111)$$

in agreement with the supergravity result (104).

## 7 Conclusions

In this work we have shown that a large class of D-brane probe actions can be recovered from a simple ansatz for the Matrix theory effective action. We subjected our ansatz to a number of highly demanding consistency checks and found the correct behavior in all cases. In the case of D0-brane sources we were able to perform the checks to all loop orders, whereas we were restricted to one loop in the case of D4-brane sources. The obstacle which prevented us from extending the latter results beyond one loop was the presence of ambiguities in evaluating the trace over the source variables. The resolution of the trace ambiguities would constitute a significant advance, and would allow us to

make contact with, for example, the discrepancies reported in [38]. One would also like to be able to treat fully non-abelian backgrounds, instead of having to assume, as was the case here, that commutators of field strengths are small.

It is worth pointing out some differences between the approach to scattering in Matrix Theory developed here and other approaches in the literature. In the majority of cases appearing elsewhere, one computes first the phase shift of the probe rather than its effective Lagrangian. Having a formalism in which the Lagrangian appears directly, as it does here, is a great advantage when one treats complicated processes involving, for instance, non-extremal branes. Computing the phase shift in such circumstances would be prohibitively difficult. Another difference with other work concerns the use of T-duality. Many treatments take the branes to be wrapped on a torus, perform T-duality, and use the relation between Matrix Theory and super Yang-Mills field theory [1, 40]. In contrast, we always work in the original spacetime. The latter approach seems to us to be simpler and to make the physical picture more readily visualizable.

It would be interesting to generalize our methods to include non-extremal sources, similar to what was done in [20]. The novel feature in this case is that one has to average over all degenerate backgrounds in order to compare with supergravity results. Perhaps Matrix theory can shed light on this intriguing situation. For instance, it is possible to include both non-extremal probes and sources into the MT effective action (17) and do at least one loop calculations.

Finally, it is suggestive that our ansatz is as simple as it is. It might be expected that, with a proper understanding, one could derive it directly from first principles without having to explicitly evaluate an infinite series of loop diagrams.

## Note Added

As this work was being completed, there appeared ref. [42] where similar ideas were developed independently.

## Appendix: Null Reduction

In this appendix we present, for convenience, the null reductions of the D0-brane and D(4+0)-brane metrics. The method we use is to start with the standard ten dimensional backgrounds, lift them up to eleven dimensions using the relation [41]

$$ds_{11}^2 = e^{-2\phi/3} ds_{10}^2 - e^{4\phi/3} (dx^{11} - dx^M C_M)^2, \quad (112)$$

and then reduce back to ten dimensions along a null direction.

## A.1 D0-brane

In ten dimensions

$$\begin{aligned} ds_{10}^2 &= H_0^{-1/2} dt^2 - H_0^{1/2} dx^i dx^i \\ e^{-\phi} &= H_0^{-3/4} \quad ; \quad C_t = H_0^{-1} - 1. \end{aligned} \quad (113)$$

Lifting to eleven dimensions we find

$$ds_{11}^2 = dt^2 - (dx^{11})^2 - h(dx^{11} - dt)^2 - dx^i dx^i \quad (114)$$

where we have defined  $h = H_0 - 1$ . Transforming to null coordinates  $x^\pm = x^{11} \pm t$ , and denoting  $\tau = x^+/2$ , the eleven dimensional metric appears as

$$\begin{aligned} ds_{11}^2 &= 2d\tau dx^- - h dx^- dx^- - dx^i dx^i \\ &= e^{-2\phi/3} ds_{10}^2 - e^{4\phi/3} (dx^- - C_\tau d\tau)^2. \end{aligned} \quad (115)$$

From the above we deduce that  $e^{-\phi} = h^{-3/4}$ ,  $C_\tau = h^{-1}$ . The null reduced metric in ten dimensions is then [24]

$$ds_{10}^2 = h^{-1/2} d\tau^2 - h^{1/2} dx^i dx^i. \quad (116)$$

The difference between this metric and the starting metric is that the harmonic function  $H_0$  has been transformed into  $h$ .

## A.2 D(4+0)-brane

The ten dimensional metric of the marginally bound (4+0) state [36] is

$$\begin{aligned} ds_{10}^2 &= H_0^{-1/2} H_4^{-1/2} dt^2 - H_0^{1/2} H_4^{-1/2} (dx_1^2 + \dots + dx_4^2) - H_0^{1/2} H_4^{1/2} (dx_5^2 + \dots + dx_9^2) \\ e^{-\phi} &= H_0^{-3/4} H_4^{1/4} \quad ; \quad C_t = H_0^{-1} - 1. \end{aligned} \quad (117)$$

Lifting to eleven dimensions gives

$$ds_{11}^2 = H_4^{-1/3} [dt^2 - (dx^{11})^2 - h(dx^{11} - dt)^2] - H_4^{-1/3} (dx_1^2 + \dots + dx_4^2) - H_4^{2/3} (dx_5^2 + \dots + dx_9^2). \quad (118)$$

Transforming to null coordinates and reducing to ten dimensions in the same way as for the D0-brane case yields

$$\begin{aligned} ds_{10}^2 &= h^{-1/2} H_4^{-1/2} d\tau^2 - h^{1/2} H_4^{-1/2} (dx_1^2 + \dots + dx_4^2) - h^{1/2} H_4^{1/2} (dx_5^2 + \dots + dx_9^2) \\ e^{-\phi} &= h^{-3/4} H_4^{1/4} \quad ; \quad C_\tau = h^{-1}. \end{aligned} \quad (119)$$

Comparing with (117), we see that the harmonic function  $H_0$  for the D0-brane has been replaced by  $h$ , while the harmonic function  $H_4$  keeps its original form.



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